# Design of an Anthropomorphic Closed-Loop Chain Manipulator 

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#### Abstract

The aim of this project is to design, analyze, and make an anthropomorphic arm that is controlled by linear actuators, forming a closed-loop kinematic chain. We present a design that is closer to workings of the human muscle by simulating different muscles such as the deltoid, bicep, and the tricep. We show the design process by using inverse kinematics on different subproblems, then do a full forward kinematic analysis including a simulation of the robot. Finally, a mechanical implementation is presented.


## I. Introduction

Many attempts have been made to apply bio-inspired concepts in designing a robotic manipulator, during which some novel ways of actuation and mathematical insights were found. The main difference between popular implementations of manipulators and the human body comes from the fact that joints in the human body are driven by muscles, forming a closed-loop kinematic chain, while manipulators are mostly direct-driven from the joints themselves.

This closed-loop construction allows for a more interesting, but difficult design with advantages and disadvantages. Some advantages of the passive construction is that it leads to more strength against dynamic load and stability. The structure may maintain tensegrity concepts to be more stable in structure as well. However, the design process for reaching an equivalent dexterity space with a servo-driven (active) manipulator is considerably more difficult.

We try to explore this problem by approximating human muscles as linear actuators. Some works we mention use artificial muscles and pneumatic and hydraulic actuators, but we focus on the ability of the linear actuators to maintain its length even when no load is applied. While this freezes the stiffness of the muscles, it allows for a safer operation of the robot.

We will first go through a design process by using inverse kinematic subproblems of each of the joints, then derive the full forward kinematics of the manipulator. Then we verify the solution by a simulator and implement the manipulator through mechanical hardware, verifying our forward kinematics solution.

## II. Previous Work

The mechanism of the human shoulder has been extensively researched on in the field of biomedical or biomechanical engineering. Lugo et al. [1] analyzes different structures of the rotator cuff, which involves various ligaments, joints, and muscle-tendon structures in order to mobilize and stabilize the movement the human arm. In particular it focuses on the roles of these muscles in view of static and dynamic stability. Rosso et al.[2] focus on the role of the deltoid
muscle in limiting the translation of the humeral joint which strengthens the fact that the human shoulder joint acts as a 3-DOF spherical joint with limited degree of translation.

While most industrial manipulators (Stanford Arm, PUMA, UR-10, etc.) implement two revolute joints in order to approximate a spherical joint, it is a well-know fact that this construction leads to singularities in the robot's workspaces. Two popular alternatives to deal with this problem are the use of Permanent-Magnet Spherical Motors [3], where a spherical joint itself is actuated by a spherically arranged array of solenoids, and a passively-driven spherical joint using parallel kinematic mechanisms such as Stewart Platforms [4]. While the former method has to simulate a perfectly spherical joint with continuous rotation, it has disadvantages in torque capacities and lacks the amount of stability provided by a closed-loop mechanism such as the latter.

In our work, we propose controlling spherical joint of a shoulder and a revolute joint of the elbow by passively driving a joint using electrical linear actuators. Passive control of joints using closed-loop mechanisms have been popular among bio-inspired manipulator designs due to its dynamic stability and the ability to handle static load more efficiently. Lenarcic and Stanisic [10] have researched into simulating a human shoulder complex by implementing a stewart platform with an additional offset angle to widen the range of motion allowed by the mechanism movement. The HUMA arm by Okadome et al. [16] implement this mechanism using pneumatic actuators and are successful in simulating a human shoulder complex. Okada and Nakamura [11] implement a rigid link design with 3DOF in a closed kinematic chain, which also successfully simulates the human shoulder joint. Recent kinematic analysis is done by Ingram et al., [13] to analyze a 3-3 Stewart Platform implementing a human shoulder.

This work focuses on adapting similar mechanisms by an electric linear actuator as opposed to pneumatic [16] or hydraulic [23] approaches. Particularly we focus on the ability of electric linear actuators to implement a worm-rackpinion construction where the movement of the pinion cannot drive the worm. Thus the manipulator has the ability to hold a load in a certain position without consuming much energy.

Additionally we focus on the ability of the anthromorphic arm to isolate the load on the end-effector from the actuators, which makes the manipulator more efficient in carrying static load compared to simple motors where the whole moment must be counteracted by the motor's torque, as well as more safe due to its ability to stop when no power is applied. The problem of static-load mitigation has traditionally been
done by methods such as gravity-counterbalancing: Whitney and Hodgins [14] implement a anthropomorphic solution to simulate a singularity-free arm with gravity counterweights. Recent interests in tensegrity has also motivated the analysis of muscloskeletal manipulators with stiffness control. Lessard et al. [6] have implemented a tensegrity-oriented actuator with isolation of compression components. This work aims to follow a similar analysis to show the structural advantage of passively-controlled manipulators.

## III. Mechanical Platform Design

We propose a passively-controlled anthropomorphic manipulator by imitating the human bone-joint structure with a kinematic chain, then passively controlling it by an array of linear actuators. This bone-joint kinematic chain is illustrted in Figure 1.


Fig. 1. (Left) Anatomy of a human arm [21] (Center) Simple kinematicchain approximation of our manipulator (Right) Linear actuators controlling for passive control of joints. Active controlled joints are colored in red.

There are 3 linear actuators whose lengths will be denoted by $\vec{S}_{1}, \vec{S}_{2}, \vec{S}_{3}$ to passively control a spherical shoulder joint. This simulates the working of the three deltoid muscles (anterior, lateral, posterior). The elbow joint is overconstrained by 2 linear actuators $\left(\vec{L}_{1}, \vec{L}_{2}\right)$ to simulate the bicep and the tricep. The wrist joint is actively controlled in circumduction by a servomotor (with $\theta_{3}$ variable) to simulate the supinator and pronator muscles, and controlled in flexsion by a sixth linear actuator $\left(\vec{L}_{3}\right)$ to simulate the flexor muscle.

While this construction of linear actuator is efficient in energy and handling static load, they inherently limit the allowed range of movement within each joint's movement due to collision and mechanical throw limits. Thus the key factor in implementing a good manipulator design is to maximize the reachable dexterity workspace of the manipulator (other issues such as dynamic stability can be discussed later). Again using an anthropomorphic analysis, research by Klopcar et al. [19] suggests that the human shoulder has $230^{\circ}$ range of movement along the sagittal plane (that divides the body into left and right), $180^{\circ}$ range of movement along the frontal plane (that divides the body into back and front), and $150^{\circ}$ range of movement along the axial plane (that divides the body into head and tail). Kurrilo et al. [1] also supports similar range of movement by evaluation using Kinect. We aim to reproduce these results, with limitations due to the fact
that the human shoulder joint is not completely spherical. The above results also utilize additional joints at the collar bone and the upper back.

## A. Design Process

For designing the mechanical construction of the manipulator we focus on individual subproblems of each joint (shoulder, elbow, wrist) and the ease of figuring out the inverse kinematics of each subproblem to decide on different parameters. Given a goal range of movements (Euler angles $\psi, \theta, \phi$ for shoulder joint and 2-dimensional angle $\theta$ for elbow and wrist joint) and linear actuator constraints $\vec{L}_{\text {min }} \leq$ $\vec{L} \leq \vec{L}_{\text {max }}$, the design problem is to select the start and end points of the linear actuators so that the goal angle range can be covered.

## B. Shoulder Joint

The shoulder joint is critical in maximizing the allowed angular displacement of the whole manipulator. Due to the fact that it is a spherical joint, the performance of a shoulder joint is evaluated by the percentage of the $\mathrm{SO}(3)$ outer manifold it can cover. While the Stewart's platform is efficient in simulating three degrees of freedom, it relatively limits the ranges of motion of the spherical joint. We solve this problem by taking the approach in The frame definitions of the shoulder joint is illustrated in Figure 2


Fig. 2. (Left) Anatomy of a Deltoid (Right) Frame Definition of Analysis
Denoting the shoulder frame as $s$ and the elbow frame as $e$, we define the center of these frames to be coincident similar to DH analysis, with rotational offset defined by ZYX Euler angles $(\psi, \theta, \phi)$. Then given our range of goal angles $(\psi, \theta, \phi)$ and the limit of linear actuators $\vec{S}_{\text {min }} \leq \vec{S} \leq \vec{S}_{\text {max }}$, our goal is to design parameters $\vec{r}_{1}^{s}, \vec{r}_{2}^{s}, \vec{r}_{3}^{s}, \vec{r}_{1}^{e}, \vec{r}_{2}^{e}, \vec{r}_{3}^{e}$. Using the ease of inverse kinematics of this construction, we can see that the vectors of linear actuators can be calculated by frame conversions:

$$
\begin{aligned}
& \vec{S}_{1}=\boldsymbol{R}_{e}^{s}(\psi, \theta, \phi) \vec{r}_{1}^{e}-\vec{r}_{1}^{s} \\
& \vec{S}_{2}=\boldsymbol{R}_{e}^{s}(\psi, \theta, \phi) \vec{r}_{2}^{e}-\vec{r}_{2}^{s} \\
& \vec{S}_{3}=\boldsymbol{R}_{e}^{s}(\psi, \theta, \phi) \vec{r}_{3}^{e}-\vec{r}_{3}^{s}
\end{aligned}
$$

where $\boldsymbol{R}_{e}^{s}(\psi, \theta, \phi)$ is a rotation matrix of the elbow frame seen from the shoulder frame in $\mathrm{SO}(3)$ as a function of ZYX

Euler angles. The dexterity space can be thus calculated by the following algorithm:

```
Algorithm 1 Dexterity Space Search
    Given \(\vec{r}_{1}^{s}, \vec{r}_{2}^{s}, \vec{r}_{3}^{s}, \vec{r}_{1}^{e}, \vec{r}_{2}^{e}, \vec{r}_{3}^{e}\) and dexterity space \(\mathscr{D}\),
    for \(\boldsymbol{R}_{e}^{s}(\psi, \theta, \phi) \in S O(3)\) do
        Calculate \(\left\|\vec{S}_{1}\right\|,\left\|\vec{S}_{2}\right\|,\left\|\vec{S}_{3}\right\|\)
        if \(\left\|\vec{S}_{\text {min }}\right\| \leq\|\vec{S}\| \leq\left\|\vec{S}_{\text {max }}\right\|\) then
            Include \(\boldsymbol{R}_{e}^{s}(\psi, \theta, \phi) \in \mathscr{D}\)
        end if
    end for
```

Figure 3 includes the result of this algorithm for design parameters $\vec{r}_{1}^{s}, \vec{r}_{2}^{s}, \vec{r}_{3}^{s}, \vec{r}_{1}^{e}, \vec{r}_{2}^{e}, \vec{r}_{3}^{e}$.


Fig. 3. Result of dexterity space search of the passively driven spherical joint

We can observe that the dexterity space covers most of the intended space of the shoulder (ideally half the cover of the $S O(3)$ manifold), with $\approx 150^{\circ}$ of cover space along a single plane. However we can also observe that the above analysis doesn't keep in mind the physical constraints of collision between different components such as linear actuator and platform, spherical joint, etc. This requires additional mathematical constraints that need to be considered into this dexterity space. Regardless the above analysis gives an insight into what design parameters will work. The design parameters for the spherical joint is listed below:

| $\vec{r}$ | $x(\mathrm{~mm})$ | $y(\mathrm{~mm})$ | $z(\mathrm{~mm})$ |
| :--- | :--- | :--- | :--- |
| $\vec{r}_{1}^{s}$ | 200 | 0 | -77.3 |
| $\vec{r}_{2}^{s}$ | -100 | 173.2051 | -77.3 |
| $\vec{r}_{3}^{s}$ | -100 | -173.2051 | -77.3 |
| $\vec{r}_{1}^{e}$ | 35 | 0 | 77.3 |
| $\vec{r}_{2}^{e}$ | -17.5 | 30.3109 | 77.3 |
| $\vec{r}_{3}^{e}$ | -17.5 | -30.3109 | 77.3 |

Added with hardware constraints of collision, the allowable angle will decrease. Works in Lenarcic and Stanisic [10] and Okadome et al. [16] attempt to solve this issue by augment the allowed angle by a constant angle-offset, which is a possible improvement for later.

## C. Elbow Joint

The kinematics of a passive revolute joint controlled by a linear actuator is a simple and well-known 3-bar linkage problem whose dynamics is extensively studied in musculoskeletal robots [17]. For design purposes we follow a frame
convention similar to Denavit-Hartenburg and define two frames intersecting at the revolute joint: one attached to the elbow frame (e) and another attached to the forearm frame (f). This frame convention is illustrated in figure


Fig. 4. Overconstrained anthropological elbow joint with two linear actuators

To simulate the biomechanics of a bicep and tricep, two linear actuators are used to overconstrain a single revolute joint. The main difference between the muscle and the electric linear actuator is that the muscle only contracts, while the linear actuator is capable of pushing back the load as well. Thus while the overconstraining construction is not necessary, it further helps in handling static and dynamic forces as well as stabilizing the manipulator in place.

We can see the inverse kinematic subproblem is very simple: that fixing the elbow frame e, our goal is to have $-180^{\circ} \leq \theta \leq-45^{\circ}$ given the constraints of $\left|\vec{L}_{\text {min }}\right| \leq|\vec{L}| \leq$ $\left|\vec{L}_{\text {max }}\right|$ for linear actuators 1 and 2 . Given $\theta, \vec{L}_{1}$ and $\vec{L}_{2}$ can be defined by

$$
\vec{L}_{1}=\boldsymbol{R}_{f}^{e}(\theta) \vec{r}_{1}^{f}-\vec{r}_{1}^{e} \quad \vec{L}_{2}=\boldsymbol{R}_{f}^{e}(\theta) \vec{r}_{2}^{f}-\vec{r}_{2}^{e}
$$

where $\boldsymbol{R}_{f}^{e}(\theta)$ is a rotation in $\mathrm{SO}(2)$ of frame f as seen from frame e. Using these calculations, we can implement the design with the following parameters:

| $\vec{r}$ | $x(\mathrm{~mm})$ | $y(\mathrm{~mm})$ |
| :--- | :--- | :--- |
| $\vec{r}_{1}^{e}$ | 31.75 | 270 |
| $\vec{r}_{2}^{e}$ | -31.75 | 235 |
| $\vec{r}_{1}^{f}$ | -31.75 | 76.2 |
| $\vec{r}_{2}^{f}$ | 0 | -38.1 |

We can observe that using these design parameters we can obtain how well they match with our constraints of $\left|\vec{L}_{\text {min }}\right|=$ 197 mm and $\left|\vec{L}_{\text {max }}\right|=347 \mathrm{~mm} \mid$ using an Actuonix 150 mm throw linear actuator. This graph is illustrated in Figure 5

We can observe that the following dimensions capture our target angle well, with an allowable range of $-180^{\circ} \leq \theta \leq$ $-50.34^{\circ}$.

## D. Wrist Joint

The human wrist is an incredibly complex system of muscle-tendon complexes that allows it to move in multiple directions. The wrist joint also differs from the shoulder joint in a sense that it is biologically not spherical. We can observe that circumduction (rotation along the forearm as an axis)


Fig. 5. Linear Actuator legnth per target angle for L1 and L2
happens closer to the elbow than it is to the wrist with lesser than $180^{\circ}$. An interesting point to note it that the rotation axis of circumduction within the human forearm lies along the Ulna (thus closer to the fourth and fifth fingers) then it is to the center of the hand. While the reasons for this is unknown, this complicates the geometry of the wrist joint significantly.

Additionally, the wrist joint is able to rotate $160^{\circ}$ in the axis parallel with the palm (flexsion and extension), with a limited range of rotation $\left(\geq 30^{\circ}\right)$ along the axis perpendicular to the palm (abduction). To simplify the movement of the wrist joint we simulate circumduction of the wrist joint by a servo (this is the only actively driven joint in our manipulator), while simulating flexsion by a linear actuator. For now abduction is left out since the movement is small in the human joint as well. The justification of using a servo motor for circumduction is that quite often, there is almost no load along the direction of circumduction.

Again we repeat a similar analysis for the linear actuator, where the forearm frame is defined as $f$ and the hand frame is defined as h . This construction is again illustrated in Figure 6.


Fig. 6. Anthropological Wrist Joint with a single linear actuator

Using a similar analysis from the elbow joint, we can see
that the length of the linear actuator can be represented by

$$
\vec{L}_{3}=\boldsymbol{R}_{h}^{f}(\theta) \vec{r}_{3}^{h}-\vec{r}_{3}^{f}
$$

where $R_{h}^{f}(\theta)$ is a rotation in $\mathrm{SO}(2)$ of frame h as seen from frame $h$. Again using the parameters, we can design using the inverse kinematic subproblem where we obtain the allowable values of $-90^{\circ} \leq \theta \leq 45^{\circ}$ under the constraint $\left|\vec{L}_{\text {min }}\right| \leq|\vec{L}| \leq\left|\vec{L}_{\text {max }}\right|$. For linear actuator 3 , we have an Actuonix 100 mm throw actuator with $\vec{L}_{\text {min }}=147 \mathrm{~mm}$ and $\vec{L}_{\text {max }}=247 \mathrm{~mm}$. This is successfully implemented by using the following parameters:

| $\vec{r}$ | $x(\mathrm{~mm})$ | $y(\mathrm{~mm})$ |
| :--- | :--- | :--- |
| $\vec{r}_{3}^{f}$ | 31.75 | -200 |
| $\vec{r}_{3}^{h}$ | 31.75 | -31.75 |

The ability of the linear actuator to track the wanted angle is illustrated in Figure 7.


Fig. 7. Linear Actuator legnth per target angle for L3

## IV. Forward Kinematics

The goal of forward kinematics of this manipulator is to express the position and orientation of the end effector as a function of actuator variables which we denote as a 6dimensional vector $\vec{\rho}_{L}=\left[d_{S 1}, d_{S 2}, d_{S 3}, d_{L 1}, d_{L 3}, \theta_{3}\right]$. $\left(d_{L 2}\right.$ is left out since it is overconstrained - $d_{L 1}$ will decide the value of $d_{L 2}$ ) It can be seen that from our definition in Figure 1, $d$ is the Euclidean norm of the corresponding vector definitions of linear actuators. While the elbow joint and the wrist joint has straightforward solutions of forward kinematics, the forward kinematics problem of the shoulder joint is a difficult problem. Thus take a serial approach to forward kinematics where the subproblem of the shoulder joint is first solved, and then the rest of the linkages are solved through DH parameters.

An interesting problem to note is that the mechanics of our manipulator controlled by linear actuators, and an equivalent manipulator controlled by servos with angular parameters are not entirely equivalent. For instance we
could have parametrized the manipulator as a 6-dimensional vector $\vec{\rho}_{A}=\left[\psi_{1}, \theta_{1}, \phi_{1}, \theta_{2}, \theta_{3}, \theta_{4}\right]$ using DH construction on the equivalent kinematic chain. (center figure in Figure 1). However, the two spaces (vector spaces of $\vec{\rho}_{L}$ and $\vec{\rho}_{A}$ ) are not isomorphic since same elements of $\vec{\rho}_{L}$ can correspond to different elements of $\vec{\rho}_{A}$ (Figure 8). Since the vector space of $\vec{\rho}_{A}$ spans the entire space allowed by our kinematic manipulator, we can see that the vector space of $\vec{\rho}_{L}$ is actually surjective to $\vec{\rho}_{A}$.


Fig. 8. Illustration of same length parameters $d_{S 1}, d_{S 2}, d_{S 3}$ corresponding to different roatations $\psi, \theta, \phi$ in the spherical joint. For instance, assuming $d_{S 1}=d_{S 2}=d_{S 3}$ will lead to a single rotation about a z-axis, $(\psi, 0,0)$. However they do not have information on if it's a positive rotation or a negative rotation as two cases are equivalent in the vector space of $\vec{\rho}_{L}$

## A. Shoulder Joint Forward Kinematics

The forward kinematic subproblem of the shoulder joint is a difficult problem and quite equivalent to the forward kinematics of the Stewart platforms with added constraints and less degrees of freedom. There are two approaches to this solution - the first one is to use geometrical constraints to derive rotation from the inverse kinematics from a rotation matrix parmetrization. Our solution goes for a Euler-angle parametrization of the rotation matrix from inverse kinematics and solving it through a non-linear optimization problem. Numerical methods of solving the forward kinematics have been discussed in [22] and [23] using optimization methods such as Newton-Raphson which is essentially a gradientdescent algorithm.

For solving the kinematics of the shoulder joint, we utilize the inverse kinematic equations of the shoulder joint which are derived from the design section.

$$
\begin{aligned}
& \vec{S}_{1}=\boldsymbol{R}_{e}^{s}(\psi, \theta, \phi) \vec{r}_{1}^{e}-\vec{r}_{1}^{s} \\
& \vec{S}_{2}=\boldsymbol{R}_{e}^{s}(\psi, \theta, \phi) \vec{r}_{2}^{e}-\vec{r}_{2}^{s} \\
& \vec{S}_{3}=\boldsymbol{R}_{e}^{s}(\psi, \theta, \phi) \vec{r}_{3}^{e}-\vec{r}_{3}^{s}
\end{aligned}
$$

The forward kinematics problem is to obtain the rotation of the shoulder joint $\psi, \theta, \phi$ given the lengths of the linear actuators $\left\|\vec{S}_{1}\right\|,\left\|\vec{S}_{2}\right\|,\left\|\vec{S}_{3}\right\|$. Thus we define a function $f$ that maps the Euler angles to the squared error term:

$$
f(\psi, \theta, \phi)=\left[\begin{array}{c}
\left(\left\|\boldsymbol{R}_{e}^{s}(\psi, \theta, \phi) \vec{r}_{1}^{e}-\vec{r}_{s}^{s}\right\|-\left\|\vec{S}_{3}\right\|\right)^{2} \\
\left(\left\|\boldsymbol{R}_{e}^{s}(\psi, \theta, \phi) \vec{r}_{2}^{e}-\vec{r}_{s}^{s}\right\|-\left\|\vec{S}_{2}\right\|\right)^{2} \\
\left(\left\|\boldsymbol{R}_{e}^{s}(\psi, \theta, \phi) \vec{r}_{3}^{e}-\vec{r}_{3}^{s}\right\|-\left\|\vec{S}_{3}\right\|\right)^{2}
\end{array}\right]
$$

Then the optimization problem is defined as the following:

$$
\text { Minimize } f(\psi, \theta, \phi) \text { subject to } \psi_{\min } \leq \psi \leq \psi_{\max }
$$

$$
\theta_{\min } \leq \theta \leq \theta_{\min }, \phi_{\min } \leq \phi \leq \phi_{\max }
$$

We use Levengerg-Marquardt [24] (Dampled Least Squares) to solve this non-linear optimization problem, which is essentially a Jacobian descent algorithm. The parameters of $\psi_{\min }, \psi_{\max }, \theta_{\min }, \theta_{\max }, \phi_{\min }, \phi_{\max }$ are selected to avoid the issue illustrated in Figure 8, which tells us that $f$ is not a convex function. Because Levengerg-Marquardt is a convex optimization algorithm that cannot deal with global minimization, this procedure is necessary. Thus given linear actuator lengths $\left\|\vec{S}_{1}\right\|,\left\|\vec{S}_{2}\right\|,\left\|\vec{S}_{3}\right\|$, we can define the homogeneous transformation in $S E(3)$ that maps the elbow frame to the stationary shoulder frame by

$$
g_{e}^{s}\left(d_{S 1}, d_{S 2}, d_{S 3}\right)=\left[\begin{array}{cc}
\boldsymbol{R}_{e}^{s}(\psi, \theta, \phi) & \overrightarrow{0} \\
\overrightarrow{0}^{T} & 1
\end{array}\right]
$$

where $(\psi, \theta, \phi)$ is the result of the optimization problem on $f(\psi, \theta, \phi)$ described above, and the ZYX Euler angle to rotation matrix is
$\boldsymbol{R}_{e}^{s}(\psi, \theta, \phi)=\left[\begin{array}{ccc}c \psi c \theta & s \psi c \theta & -s \theta \\ c \psi s \phi s \theta-s \psi c \phi & c \psi c \phi+s \psi s \phi s \theta & c \theta s \phi \\ s \psi s \phi+c \psi c \phi s \theta & s \psi c \phi s \theta-c \psi s \phi & c \phi c \theta\end{array}\right]$

## B. Elbow and Wrist Forward Kinematics

For the wrist and elbow, we can see that the relation between the angle parameters of the joint and the length parameters of the linear actuator are bijective within allowed angles (Figure 5 and 7) In fact, they were designed to be bijective! Thus a DH parametrization is allowed on the joints.

To elaborate on this matter, we can mathematically see that the inverse kinematics problems in the form of

$$
d=\left\|\boldsymbol{R}_{b}^{a}(\theta) \vec{r}_{1}^{b}-\vec{r}_{1}^{a}\right\|
$$

can be formulated in a forward kinematics problem of solving $\theta$ given $d$. Squaring the terms give us an equation in the form of

$$
\begin{gathered}
A \cos ^{2} \theta+B \sin ^{2} \theta+C \cos \theta \sin \theta \\
+D \cos \theta+E \sin \theta+F=0
\end{gathered}
$$

which can then be solved by imposing real and angle range constraints. The terms are illustrated in the appendix. A similar argument can be made geometrically by observing Figure 4 and 6 - once the length is determined, the angle is determined as well.

Thus it is possible to use DH parametrization after converting linear actuator lengths to angles to calculate the forward kinematics. We use a DH construction on the elbow and wrist manipulator starting from the shoulder frame. The axis definition and frame definitions are illustrated in Figure 9.

Following DH parametrization, we can see that the forward and kinematics of the elbow and wrist manipulator can be expressed using the following parameters:


Fig. 9. Axis and Frame Definition for Elbow and Wrist Manipulator

| $i$ | $g$ | $a_{i}$ | $\alpha_{i}$ | $d_{i+1}$ | $\theta_{i+1}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | $g_{1}^{e}$ | 0 | $-\frac{\pi}{2}$ | 0 | $\theta_{1}$ |
| 1 | $g_{2}^{1}$ | $a_{1}$ (constant) | $\frac{\pi}{2}$ | $d_{2}$ (constant) | $\theta_{2}$ |
| 2 | $g_{3}^{2}$ | 0 | $-\frac{\pi}{2}$ | 0 | $\theta_{3}$ |
| 3 | $g_{T}^{3}$ | $a_{3}$ (constant) | 0 | 0 | 0 |

From these we can see that the elbow manipulator to the tool frame can be defined as

$$
g_{T}^{0}=g_{1}^{0} g_{2}^{1} g_{3}^{2} g_{T}^{3}
$$

where the transformations are functions of $a_{i}, \alpha_{i}, d_{i+1}, \theta_{i+1}$ :

$$
\begin{gathered}
g_{i+1}^{i}\left(a_{i}, \alpha_{i}, d_{i+1}, \theta_{i+1}\right) \\
=\left[\begin{array}{cccc}
\cos \theta_{i+1} & -\sin \theta_{i+1} & 0 & a_{i} \\
\sin \theta_{i+1} \cos \alpha_{i} & \cos \theta_{i+1} \cos \alpha_{i} & -\sin \alpha_{i} & -d_{i+1} \sin \alpha_{i} \\
\sin \theta_{i+1} \sin \alpha_{i} & \cos \theta_{i+1} \sin \alpha_{i} & \cos \alpha_{i} & d_{i+1} \cos \alpha_{i} \\
0 & 0 & 0 & 1
\end{array}\right]
\end{gathered}
$$

Here we can see that $\theta_{1}$ is a function of $d_{L 1}, \theta_{2}$ is a direct manipulator variable, and $\theta_{3}$ is a function of $d_{L 3}$ ( $d_{L 2}$ is overconstrained). Thus we can see that the elbow and wrist forward kinematics can be expressed by

$$
g_{T}^{0}\left(d_{L_{1}}, \theta_{2}, d_{L_{3}}\right)=g_{1}^{0}\left(\Theta_{1}\left(d_{L_{1}}\right)\right) g_{2}^{1}\left(\theta_{2}\right) g_{3}^{2}\left(\Theta_{3}\left(d_{L_{2}}\right)\right) g_{T}^{3}
$$

where $\Theta(d)$ maps the linear actuator variable to angle DH parameter.

## C. Manipulator Forward Kinematics

With the above results we define the forward kinematics of the whole manipulator by

$$
\begin{gathered}
g_{T}^{s}\left(d_{S_{1}}, d_{S_{2}}, d_{S_{3}}, d_{L_{1}}, \theta_{2}, d_{L_{3}}\right) \\
=g_{T}^{s}\left(d_{S 1}, d_{S 2}, d_{S 3}\right) g_{0}^{e} g_{T}^{0}\left(d_{L_{1}}, \theta_{2}, d_{L_{3}}\right)
\end{gathered}
$$

Here there is one added homogeneous transformation $g_{0}^{e}$ which maps the frame $e$ at the base of the elbow to the 0 which is on top of the elbow. We can define them as frames with same orientation with offset of length of forearm. which we denote as $d_{0}$ :

$$
g_{0}^{e}=\left[\begin{array}{cc}
\boldsymbol{I} & d_{0} \hat{z} \\
\overrightarrow{0} & 1
\end{array}\right]
$$

## V. Simulation

In order to verify the forward kinematics we implement a simulation via MATLAB to graphically illustrate the position of the linkages and the linear actuators given the forward kinematics parameters $d_{S_{1}}, d_{S_{2}}, d_{S_{3}}, d_{L_{1}}, \theta_{2}, d_{L_{3}}$. Some of the
example cases are illustrated in this section. The code of the simulator is included in appendix.

## A. Maximum Shoulder Angle

In this illustration we describe the maximum shoulder angle by setting $d_{S_{2}}$ and $d_{S_{3}}$ to minimum angles of the linear actuator lengths. In this case the variables that are used are as follows:

| $d_{S 1}$ | $d_{S 2}$ | $d_{S 3}$ | $d_{L 1}$ | $\theta_{L 2}$ | $d_{L 3}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 286 | 197 | 197 | 260 | 0 | 180 |

The units are in mm. Given these variables, the resulting configuration of the manipulator is illustrated in Figure 10


Fig. 10. Illustration of Maximum Shoulder Angle Simulation. Mechanical links are illustrated in black, and frame coordinates are illustrated in green. The linear actuators are illustrated in red.


Fig. 11. Illustration of maximum shoulder displacement
From the sideview illustration in Figure 11, we can observe that the actual dexterity space is considerably smaller compared to our original design intent due to collision issues. In fact this particular spherical joint is only able to achieve a $45^{\circ}$ offset from the $z$-axis of the stationary frame which limits our dexterity space and movement.

## B. Shoulder Circumduction

We also illustrate the simulation's capability to express rotations along the axis of the forearm by setting $d_{S_{1}}, d_{S_{2}}, d_{S_{3}}$. The parameters for this simulation are

| $d_{S 1}$ | $d_{S 2}$ | $d_{S 3}$ | $d_{L 1}$ | $\theta_{L 2}$ | $d_{L 3}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 260 | 260 | 260 | 310 | $\frac{\pi}{2}$ | 200 |

The simulation result is illustrated in Figure 12. We can observe the validity of the forward kinematic solution in expressing the rotation along the $z$-axis.


Fig. 12. Illustration of shoulder platform rotation along the $z$-axis

## VI. Inverse Kinematics

The inverse kinematics of this manipuluator can be obtained by solving the inverse kinematic problem on the simple kinematic chain approximation (Figure 1 center figure) because the variable space of our linear actuator manipulator is surjective to the variable space of the equivalent servocontrolled manipulator. The inverse-kinematic of an anthropomorphic human arm with 7-DOF is a well-known problem and has been solved in closed form [28][29]. Thus without going through the full inverse kinematics derivation, we describe a methodology of switching the obtained solution in terms of angles to linear actuator lengths. This problem is trivial and was solved in the design section of this work, but the formal formulation follows:

Given goal orientation and position in homogeneous transformation $g_{T}^{s}$, the goal is to obtain parameters $\vec{\rho}_{L}=$ $\left[d_{S_{1}}, d_{S_{2}}, d_{S_{3}}, d_{L_{1}}, \theta_{2}, d_{L_{3}}\right]$. Assuming we can obtain parameters $\vec{\rho}_{A}=\left[\psi, \theta, \phi, \theta_{1}, \theta_{2}, \theta_{3}\right]$ from the original inverse kinematics formulation, we can translate it by the equations given in the design process:

$$
\begin{gathered}
d_{S_{1}}=\left\|\boldsymbol{R}_{e}^{s}(\psi, \theta, \phi) \vec{r}_{1}^{e}-\vec{r}_{1}^{s}\right\| \\
d_{S_{2}}=\left\|\boldsymbol{R}_{e}^{s}(\psi, \theta, \phi) \vec{r}_{2}^{e}-\vec{r}_{2}^{s}\right\| \\
d_{S_{3}}=\left\|\boldsymbol{R}_{e}^{s}(\psi, \theta, \phi) \vec{r}_{3}^{e}-\vec{r}_{3}^{s}\right\| \\
d_{L_{1}}=\left\|\boldsymbol{R}\left(\theta_{1}\right) \vec{r}_{1}^{f}-\vec{r}_{1}^{e}\right\| \\
\theta_{2}=\theta_{2} \\
d_{L_{3}}=\left\|\boldsymbol{R}\left(\theta_{3}\right) \vec{r}_{3}^{h}-\vec{r}_{3}^{f}\right\|
\end{gathered}
$$

Here $\boldsymbol{R}_{e}^{s}(\psi, \theta, \phi)$ is a rotation matrix in $\mathrm{SO}(3)$ and $\boldsymbol{R}(\theta)$ is a rotation matrix in $\mathrm{SO}(2)$.

## VII. Mechanical Implementation

After figuring out the necessary geometrical parameters of the manipulator, we implement a mechanical design of the passively-driven manipulator. The hardware implementation is shown in Figure 13 and submitted. The configuration of the hardware was modeled after the simulation result in Figure 10, which is maximum angle of the shoulder. One interesting


Fig. 13. Mechanical construction of the manipulator
problem that we noticed in the mechanical design was that three actuators of the spherical joint did not constrain the axial movement (circumduction) completely. This may be a result of some components in the small spherical joints above and below the linear actuators not being completely restrained, thus worsening the non-uniqueness of the forwardkinematics solution mentioned above.

## VIII. Conclusion \& Future Works

In this work we have designed the bio-inspired anthropomorphic manipulator that is passively driven by 6 linear actuators and a servo, providing 6-DOF movement. The forward kinematics i s derived with a mixed-approach of non-linear optimization and DH parameters, and validated on a simulation. Finally, the arm is mechanically constructed.

The mechanical design process was challenging as it is difficult to model all the collisions and physical parameters of the manipulator with an exact mathematical formulation. This describes the reason between the dexterity space search in Figure 3, and the actual maximum joint angle allowed by the mechanical design illustrated in Figure 11. For a joint approach will be needed between a this a Computer-Aided Design (CAD) model and the kinematic design equations.

For future work we wish to investigate the exact benefits of using a passively-controlled manipulator by carrying out static and dynamic load analysis on the model of our manipulator. Furthermore, control and dynamics of the manipulator using Jacobian and optimization for motion-planning (path and velocity) is required before the arm can be put into action.

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